

Master Thesis

# Statistical Properties of Particle Spreading in Quenched Random Media

Shiraz University

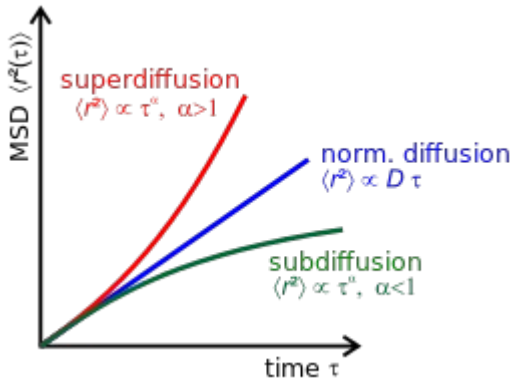
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# Motivation

## Anomalous is Normal

Klafter and I. M. Sokolov, Anomalous diffusion spreads its wings, Physics world, vol. 18, no. 8, p. 29, 2005

$$\langle x^2 \rangle \sim t^\alpha$$



CTRW

fBm

Levy flight

## Disordered Media Shows Anomaly

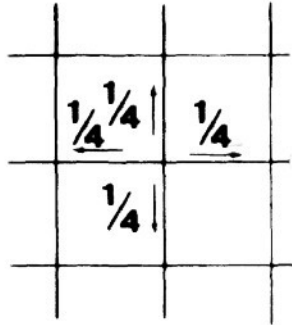
Porous Media

Cell Environment

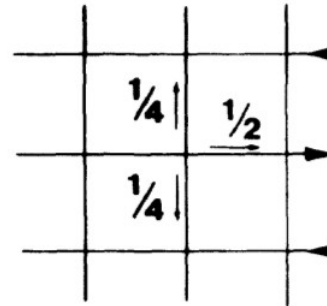
Complex Networks

...

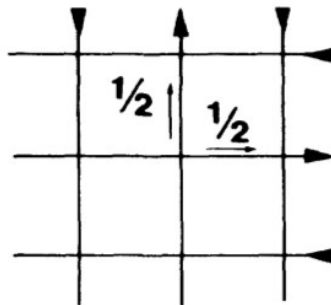
# Manhattan Grid



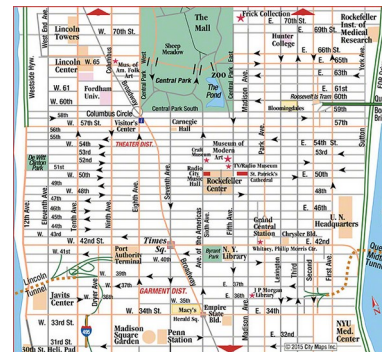
$$\alpha = 1$$



$$\alpha = 3/2$$



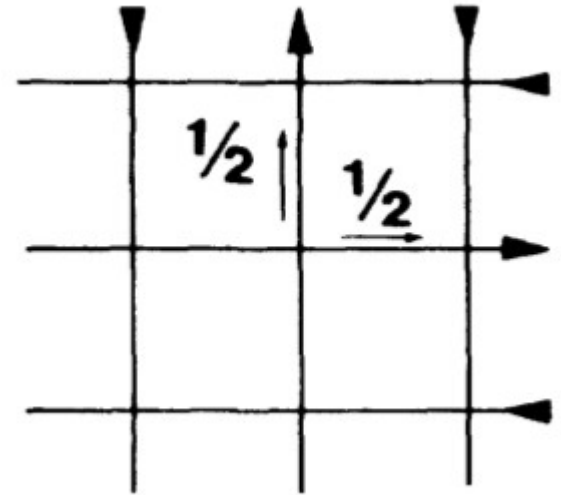
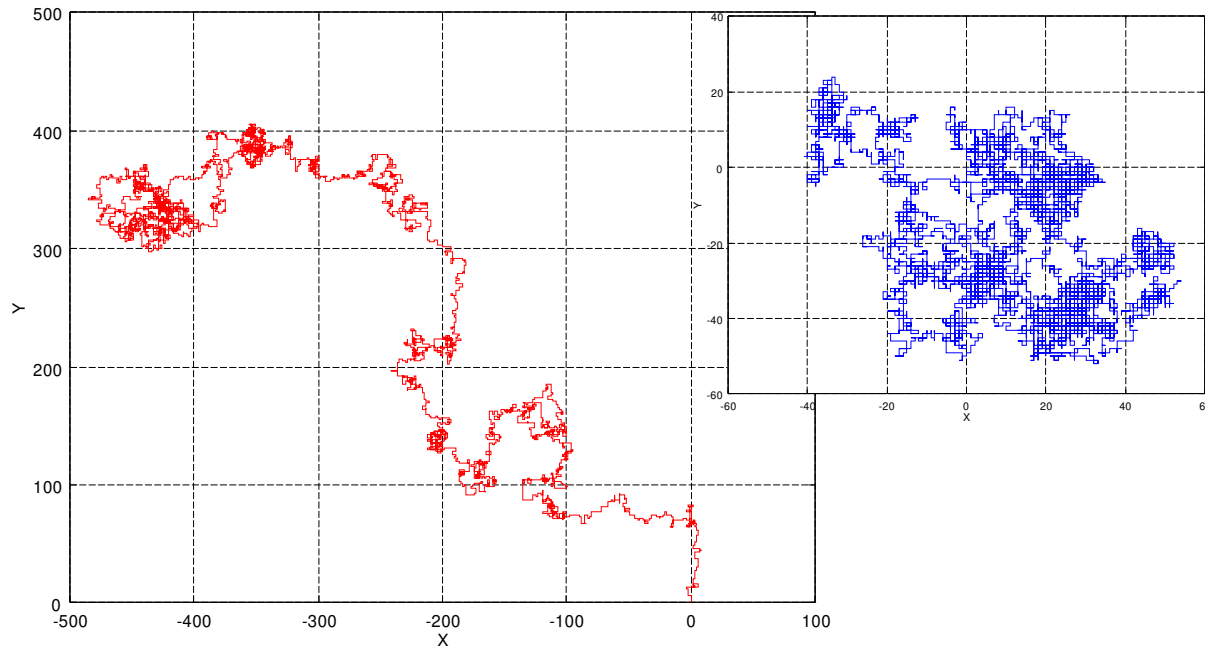
$$\alpha = 4/3$$



# Diffusion in Quenched Manhattan Grid

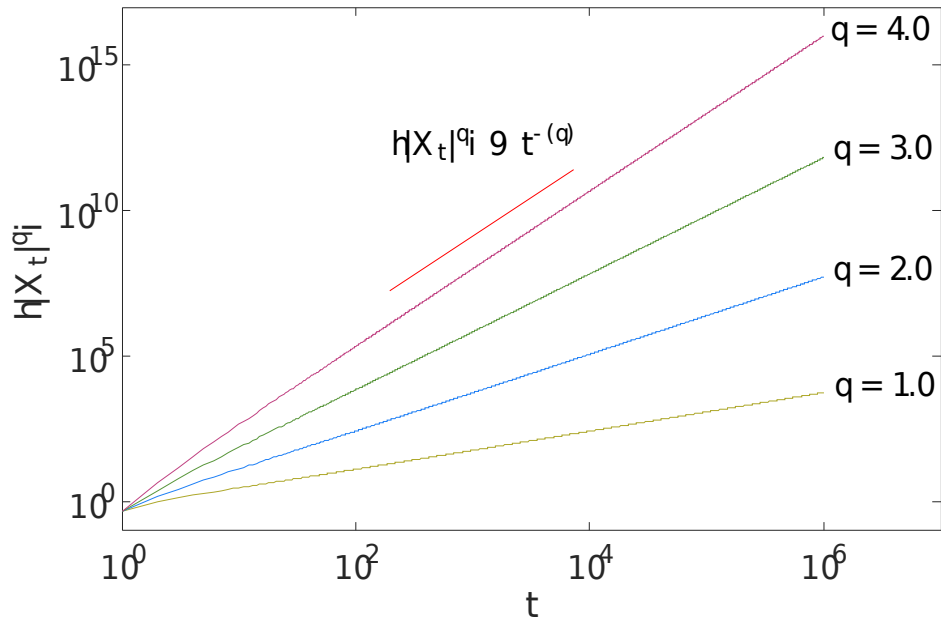
$$x(t + \Delta t) = x(t) + \eta(t)\mathcal{U}(y(t))\Delta t$$

$$y(t + \Delta t) = y(t) + [1 - \eta(t)]\mathcal{V}(x(t))\Delta t$$



The motion is Super-diffusive with  $\alpha = 1.33 \pm 0.01$

# Moments of Diffusion

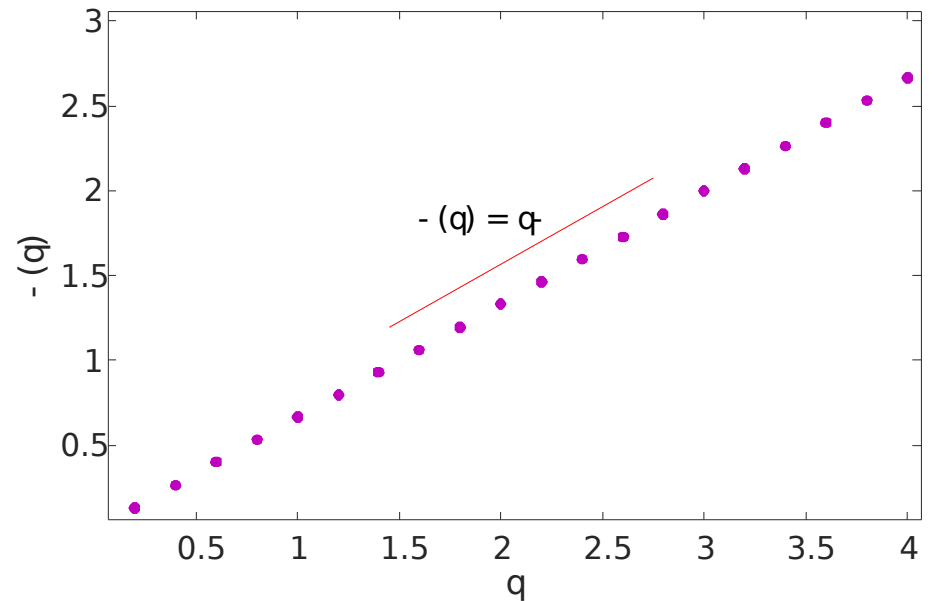


$$\langle |x|^q \rangle \sim t^{\beta(q)}$$

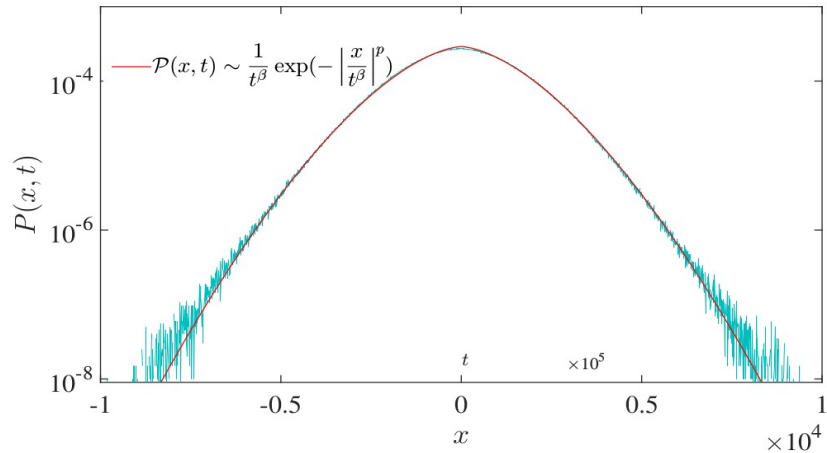
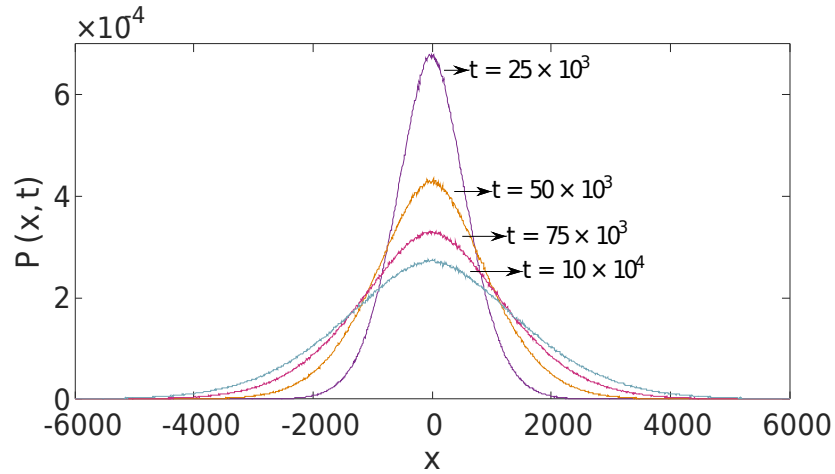
$$\beta(q) \sim q\beta$$

Self affinity exists in the process

$$\beta = 0.6667 \pm 0.0005$$



# Gaussian or Non-Gaussian



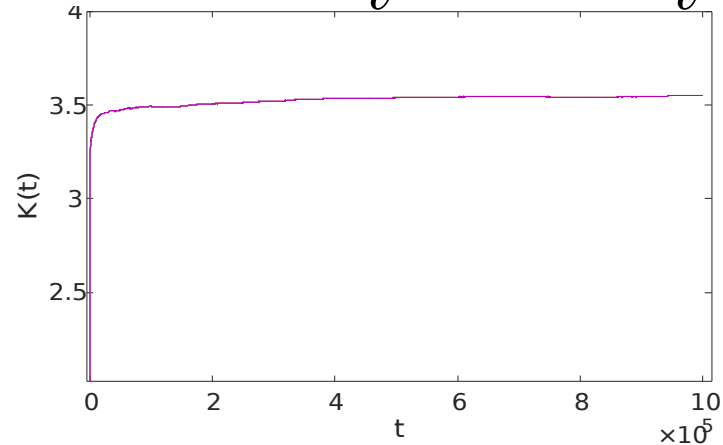
We expect  $\mathcal{P}(x, t) \sim \frac{1}{t^\beta} \mathcal{F}\left(\frac{x}{t^\beta}\right)$

$$\mathcal{K} = \frac{\langle |x(t)|^4 \rangle}{\langle |x(t)|^2 \rangle^2} = 3.54 \pm 0.01$$

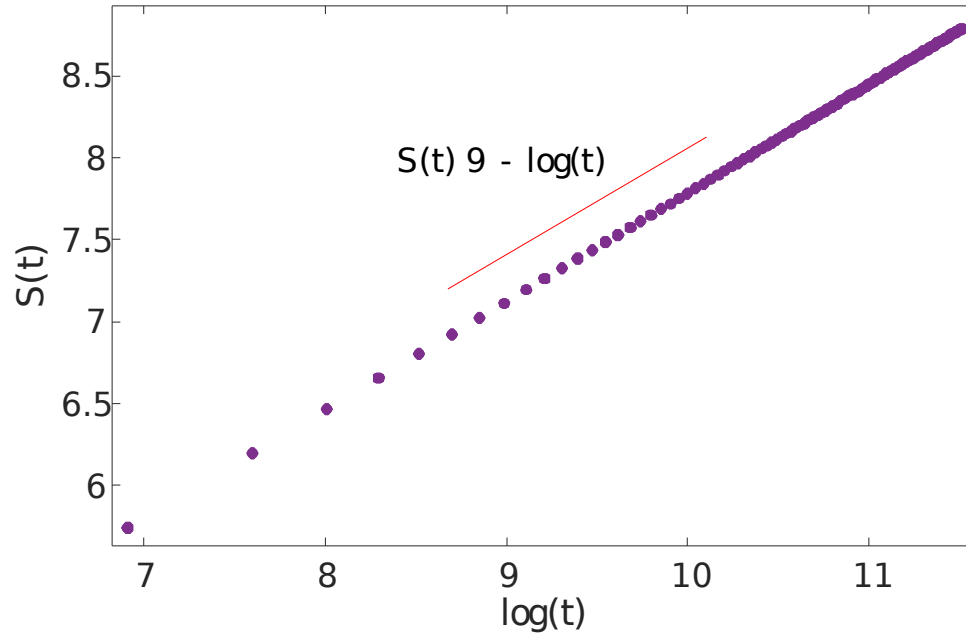
we choose GGD:  $\mathcal{F}(u) \propto \exp(-|u|^p)$

$$\mathcal{K} = \frac{\Gamma(5/p)\Gamma(1/p)}{\Gamma^2(3/p)} \implies p = 1.607 \pm 0.006$$

$$\mathcal{P}(x, t) \sim \frac{1}{t^\beta} \exp\left(-\left|\frac{x}{t^\beta}\right|^p\right)$$



# DEA

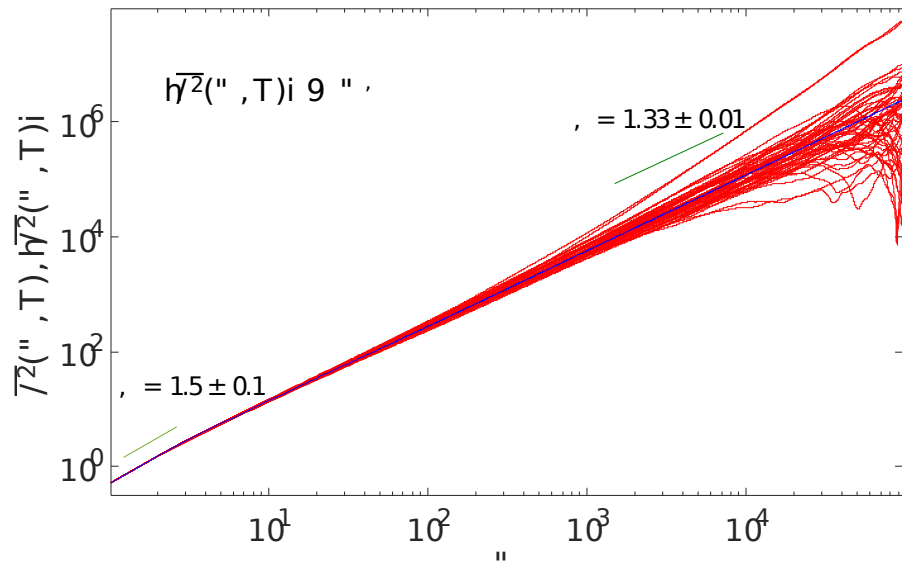


$$S(t) = - \int \mathcal{P}(\vec{r}, t) \log \mathcal{P}(\vec{r}, t) d\vec{r}$$

$$S(t) = \beta \log t - \int \mathcal{F}(u) \log \mathcal{F}(u) du = \beta \log t + \text{cnst.}$$

$$\beta = 0.6629 \pm 0.0001$$

# Weak Ergodicity Breaking and Aging



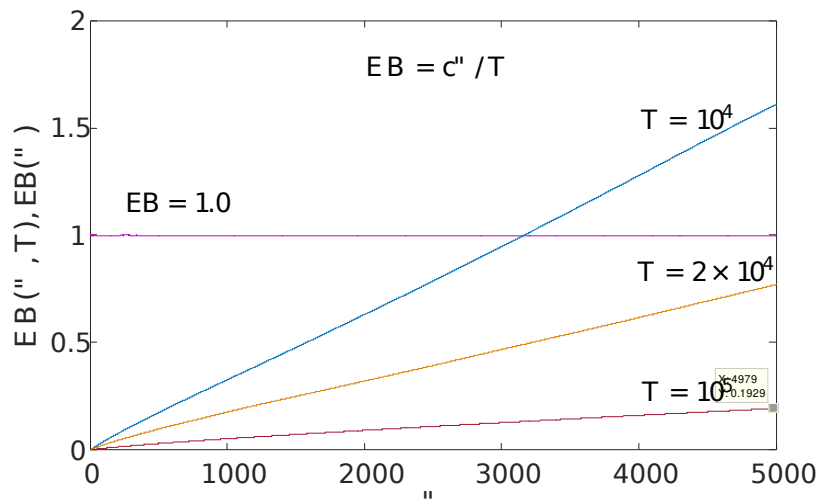
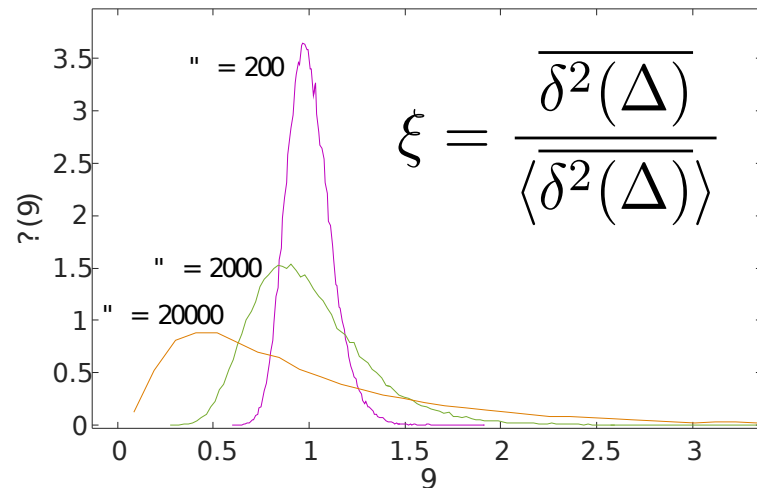
$$\overline{\delta^2(\Delta, T)} = \frac{1}{T - \Delta} \int_0^{T-\Delta} [\vec{r}(t + \Delta) - \vec{r}(t)]^2 dt$$

$$\lim_{T \rightarrow \infty} \overline{\delta^2(\Delta, T)} = \langle \vec{r}^2(\Delta) \rangle$$

$$EB(\Delta) = \lim_{T/\Delta \rightarrow \infty} \langle \xi^2 \rangle - 1$$

$$\langle \overline{\delta^2(\Delta, T)} \rangle = \frac{1}{N} \sum_{i=1}^N \overline{\delta^2(\Delta, T)}$$

$$\mathcal{EB} = \frac{\langle \overline{\delta^2(\Delta)} \rangle}{\langle \vec{r}^2(\Delta) \rangle}$$





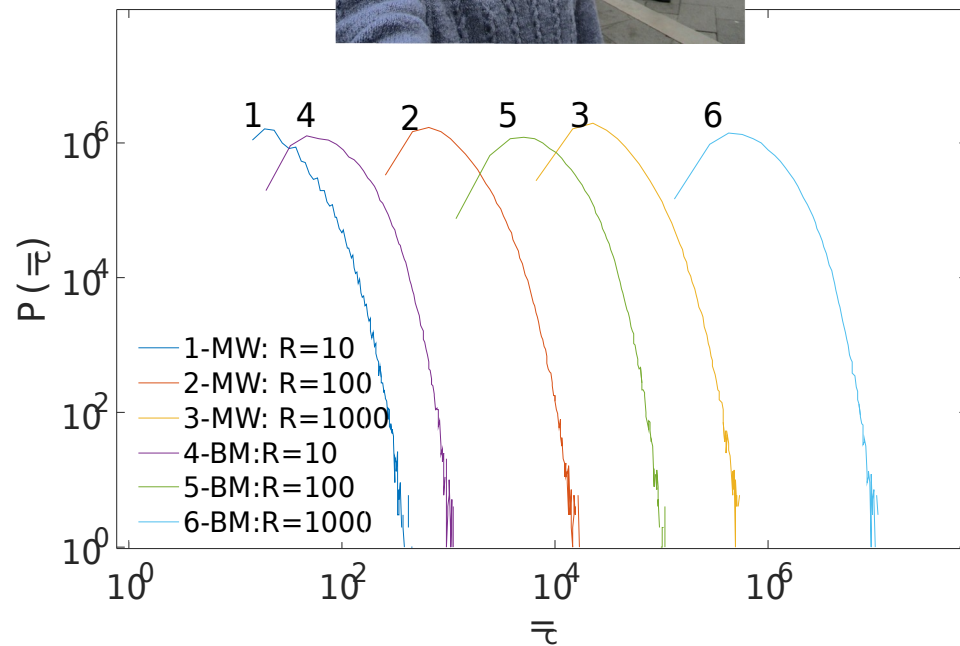
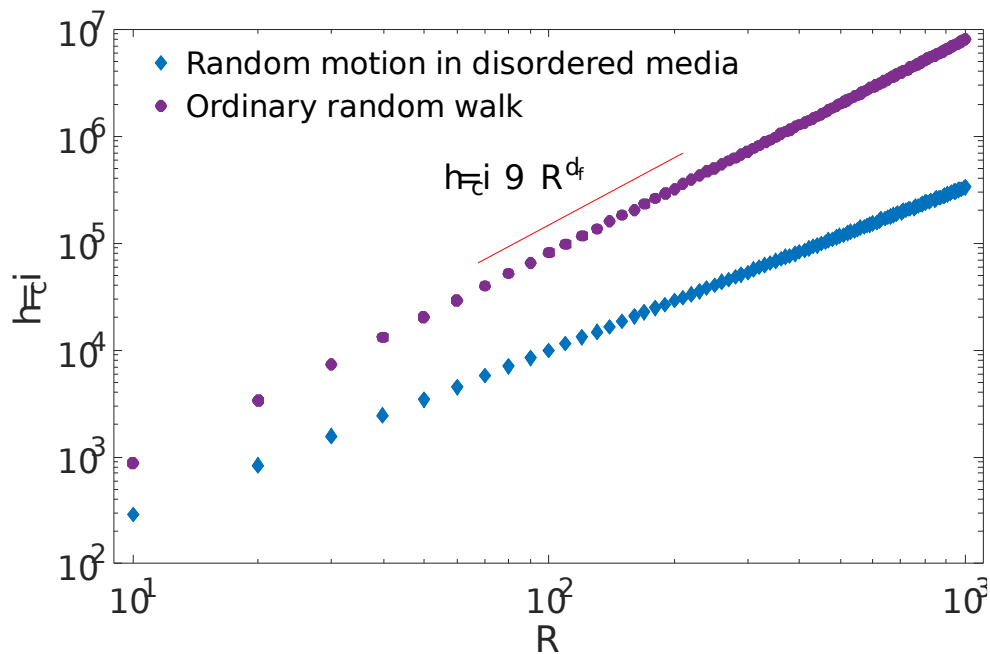
# First Exit Time

$$\tau_c = \inf\{t > 0 : |\vec{r}(t)| \geq R\}$$

$$\langle \tau_c \rangle \sim R^{d_f}$$

$$d_f = 1.522 \pm 0.005$$

$$d_f = 1.998 \pm 0.005$$



# Random Dance

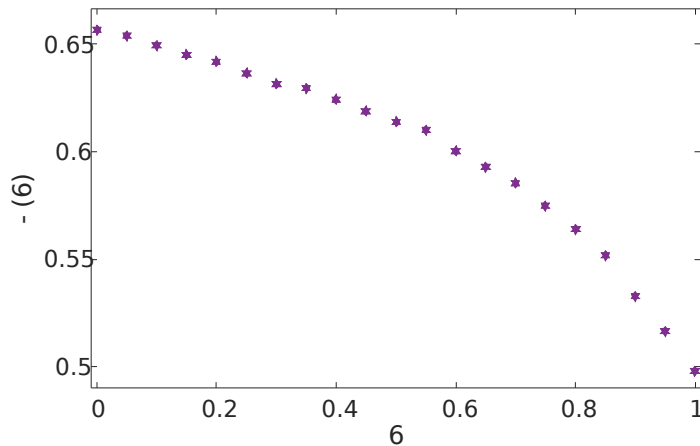
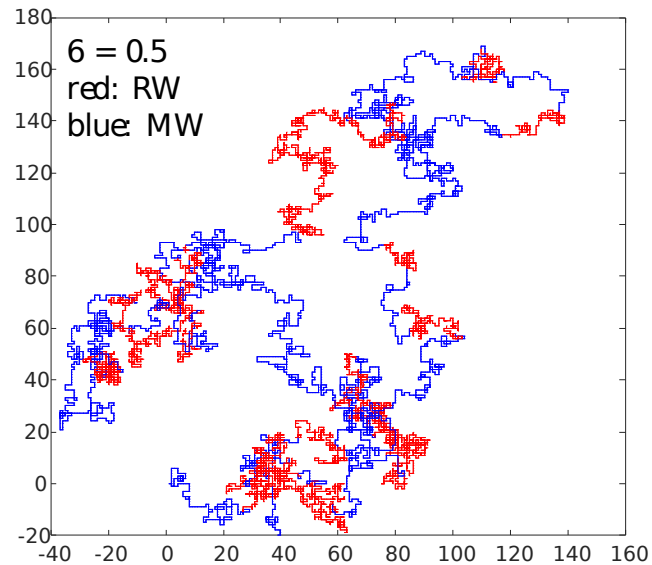
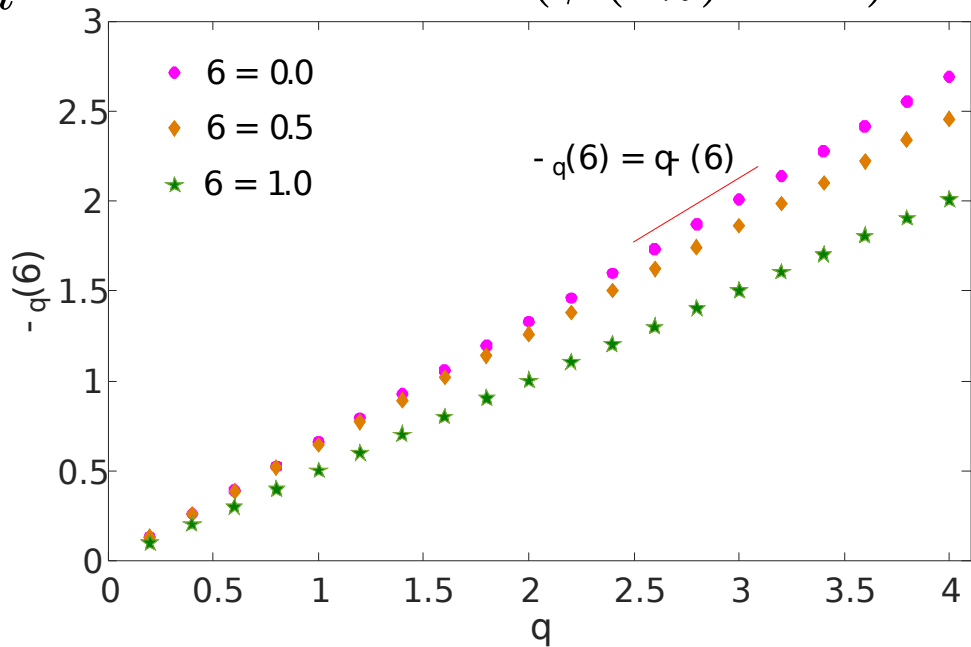
$$x(t + \Delta t) = x(t) + \left[ \phi(\tau_n)\xi(t) + (1 - \phi(\tau_n))\{\eta(t)\mathcal{U}(y(t))\} \right] \Delta t$$

$$y(t + \Delta t) = y(t) + \left[ \phi(\tau_n)\xi(t) + (1 - \phi(\tau_n))\{(1 - \eta(t))\mathcal{V}(x(t))\} \right] \Delta t$$

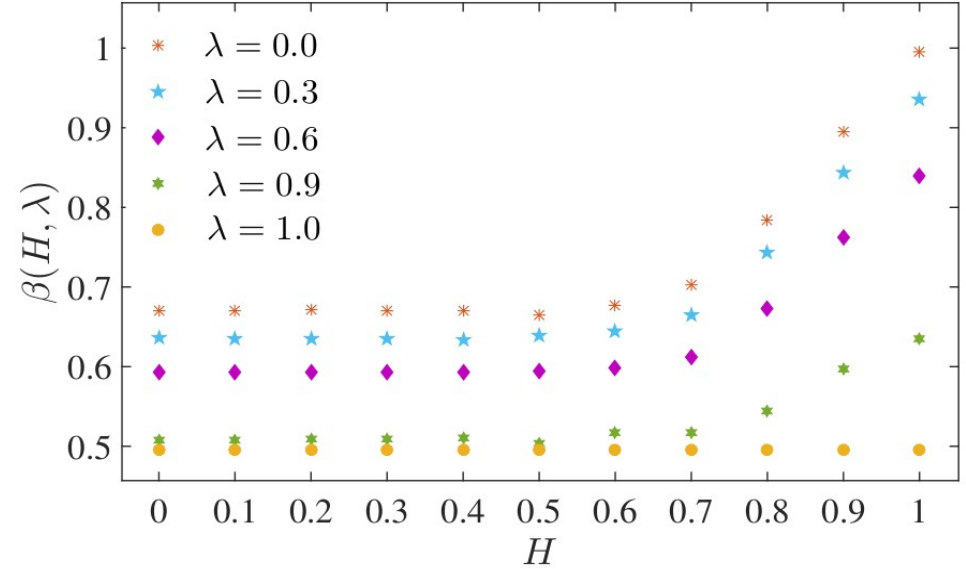
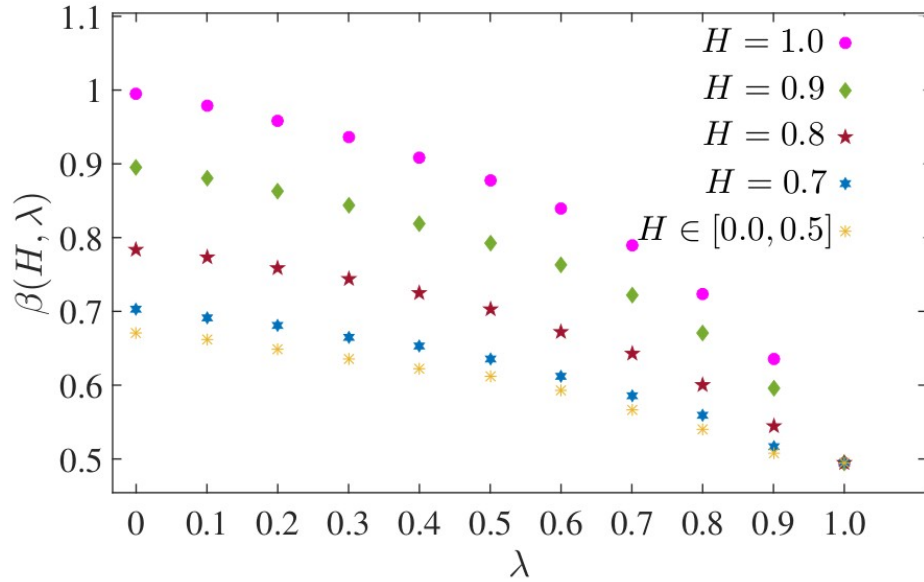
$$\sum_n \tau_n = T$$

$$P(\phi(\tau_n) = 1) = \lambda$$

$$P(\phi(\tau_n) = 0) = 1 - \lambda$$



# Correlation Effect

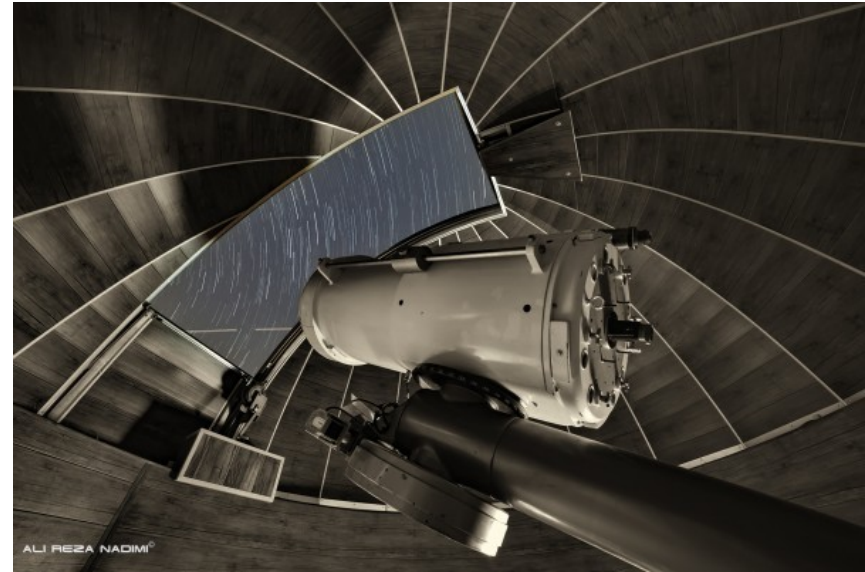
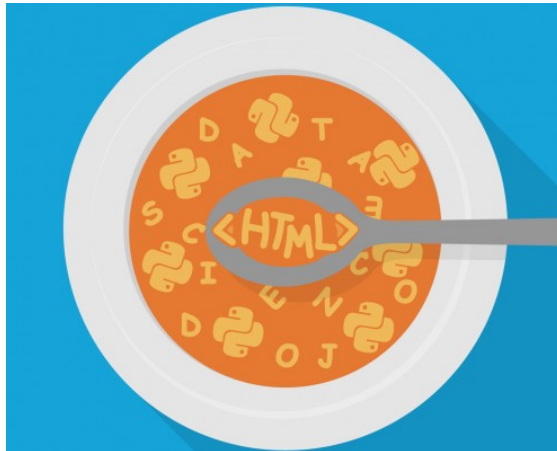


$$\langle \mathcal{U}_x[y_1] \mathcal{U}_x[y_2] \rangle = \sigma^2 [H(2H - 1)] |y_1 - y_2|^{2H-2}$$

$$\langle \mathcal{U}_y[x_1] \mathcal{U}_y[x_2] \rangle = \sigma^2 [H(2H - 1)] |x_1 - x_2|^{2H-2}$$

# Previous Projects

- Synchronization
- Variable Star Photometry
- Internship as Data Scientist





# Current Project

## WIKIPEDIA

The screenshot shows the Wikipedia article for Denny Hamlin. At the top, there is a notice: "This biographical article needs additional citations for verification. Please help by adding reliable sources. Contentious material about living persons that is uncited or poorly sourced must be removed immediately, especially if potentially libelous or harmful." Below this, a brief bio states: "James Dennis Alan 'Denny' Hamlin (born November 18, 1989) is an American NASCAR race car driver. He currently drives the No. 11 FedEx Toyota for Joe Gibbs Racing in the Sprint Cup Series and the No. 18 Toyota in the Nationwide Series. Hamlin was born in Tampa, Florida, but lived in Chesterfield, Virginia for most of his childhood. He began racing go-karts at the age of 7. Afterward, he worked his way up to NASCAR." The article also includes a section for "DNA" with a sub-section for "Laryngotracheobronchitis" (Croup). The DNA section includes a definition, symptoms, and treatment information. A table of contents is visible on the left side of the page.



The screenshot shows the Wikipedia article for "Croup". It includes a definition: "Croup, also known as laryngotracheobronchitis, is a type of respiratory infection that is usually caused by a virus." It lists symptoms such as "barking" cough, stridor, and hoarseness. The article also mentions that croup is typically diagnosed based on signs and symptoms and is usually treated with a single dose of steroids by mouth. A table of contents is visible on the left side of the page.

