Master Thesis

Statistical Properties of Particle Spreading in Quenched Random Media

Shiraz University

Navid Mousavi

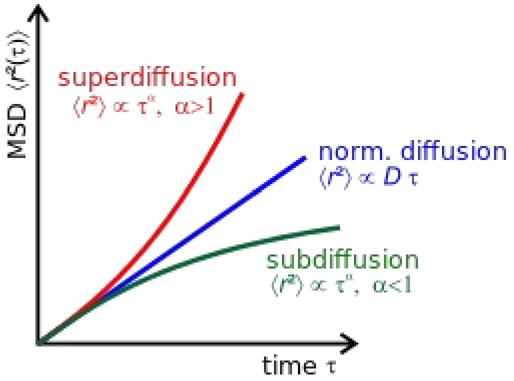
September 2019

Motivation

Anomalous is Normal

Klafter and I. M. Sokolov, Anomalous diffusion spreads its wings, Physics world, vol. 18, no. 8, p. 29, 2005

$$\langle x^2 \rangle \sim t^\alpha$$

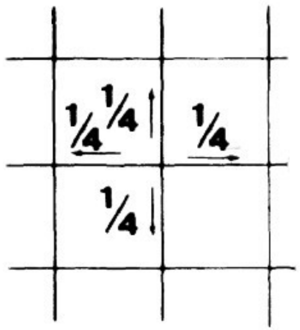


CTRW   fBm   Levy flight

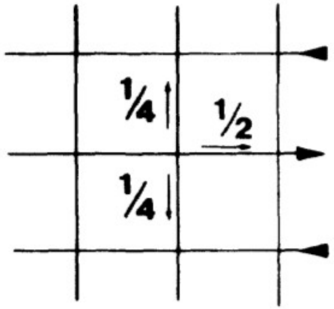
Disordered Media Shows Anomaly

Porous Media   Cell Environment   Complex Networks   ...

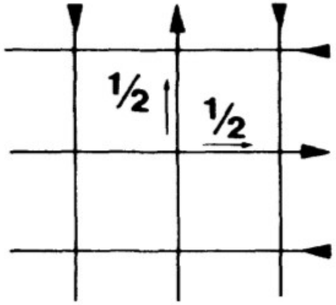
Manhattan Grid



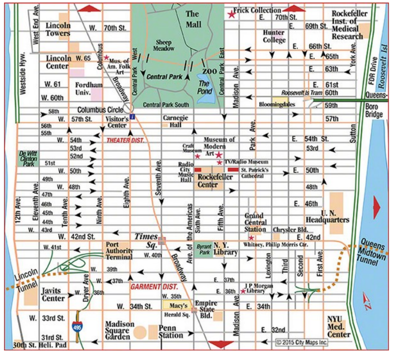
$\alpha = 1$



$\alpha = 3/2$



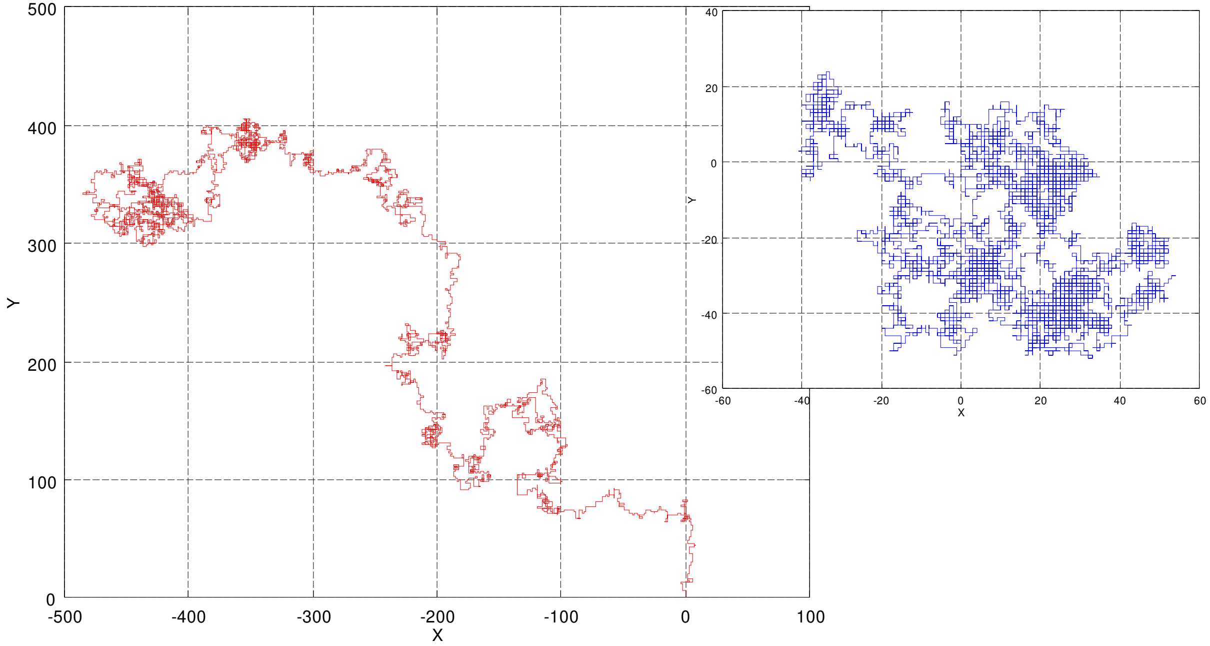
$\alpha = 4/3$

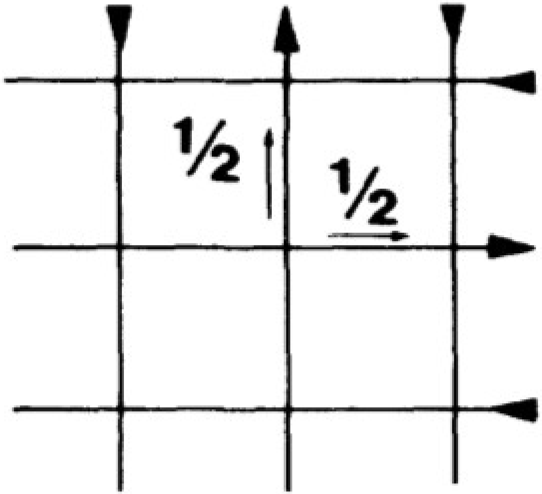


Diffusion in Quenched Manhattan Grid

$$x(t+\Delta t) = x(t) + \eta(t)\mathcal{U}(y(t))\Delta t$$

$$y(t+\Delta t) = y(t) + [1-\eta(t)]\mathcal{V}(x(t))\Delta t$$

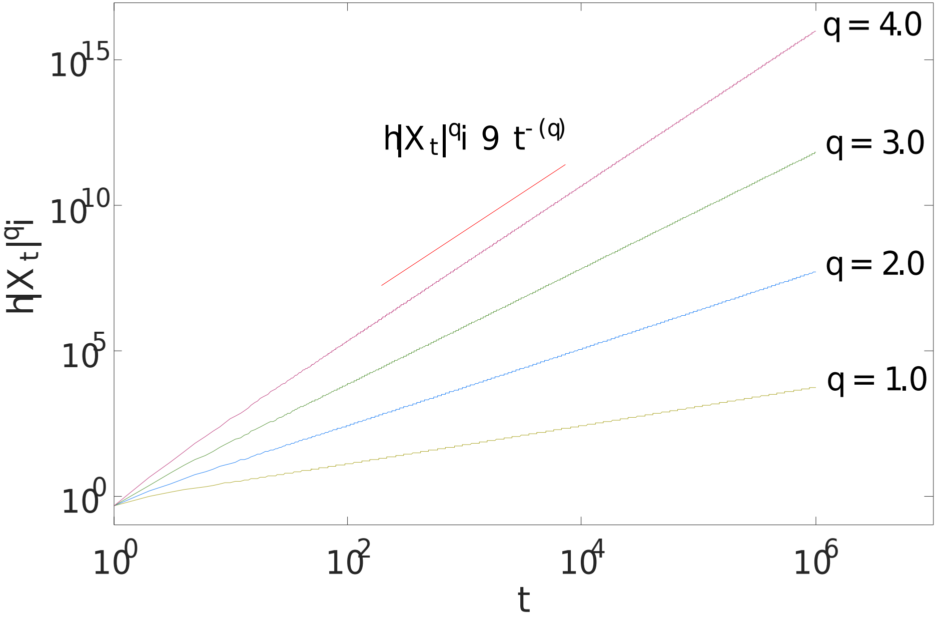




The motion is Super-diffusive with

$\alpha = 1.33 \pm 0.01$

Moments of Diffusion

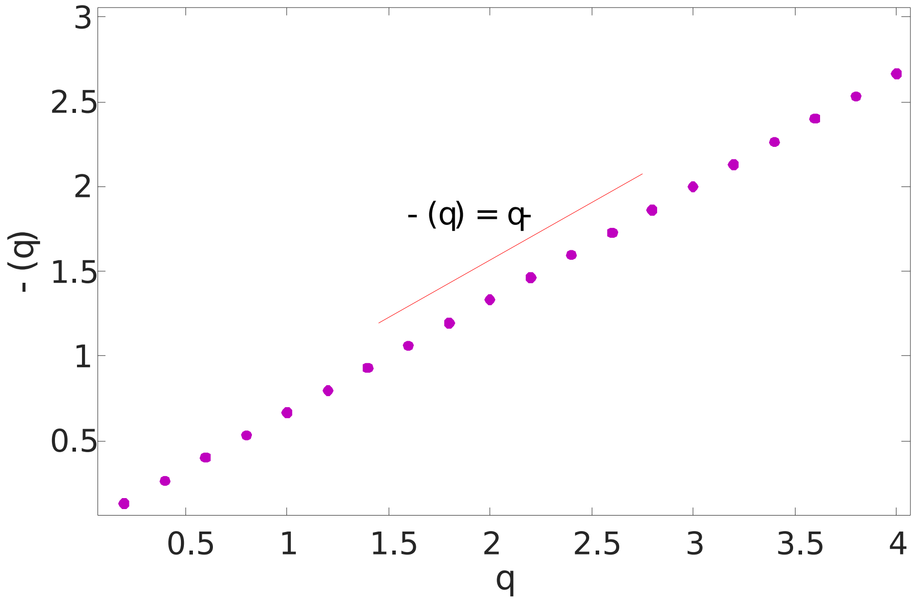


$$\langle |x|^q \rangle \sim t^{\beta(q)}$$

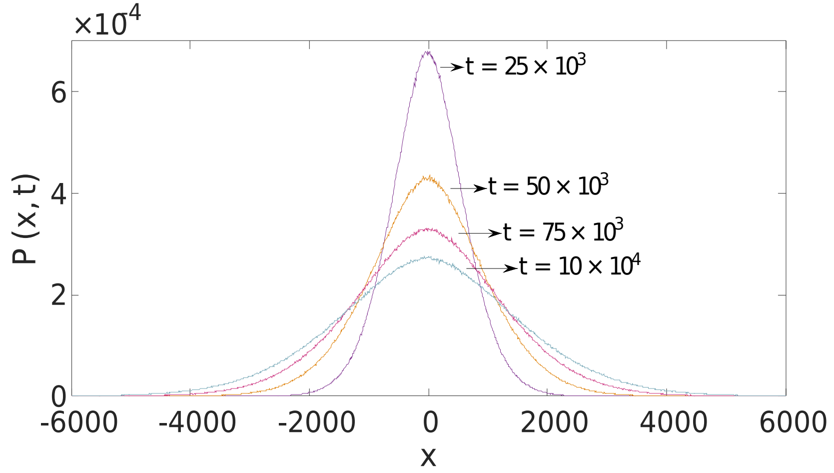
$$\beta(q) \sim q\beta$$

Self affinity exists in the process

$$\beta = 0.6667 \pm 0.0005$$



Gaussian or Non-Gaussian



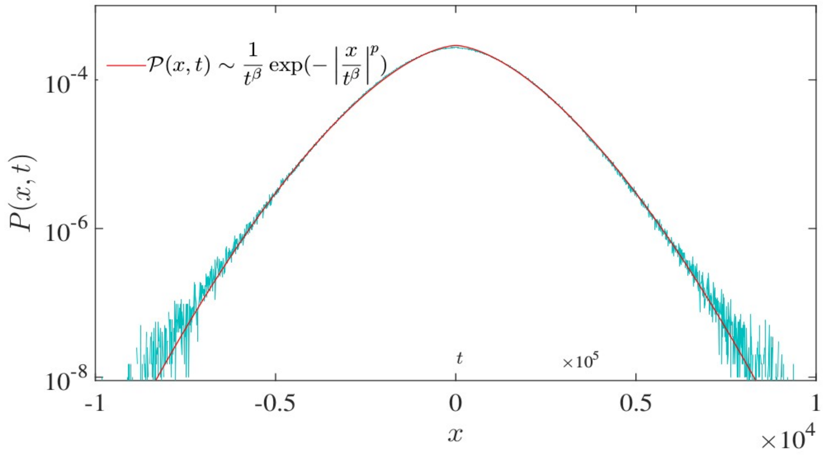
We expect $\mathcal{P}(x,t) \sim \frac{1}{t^{\beta}} \mathcal{F}(\frac{x}{t^{\beta}})$

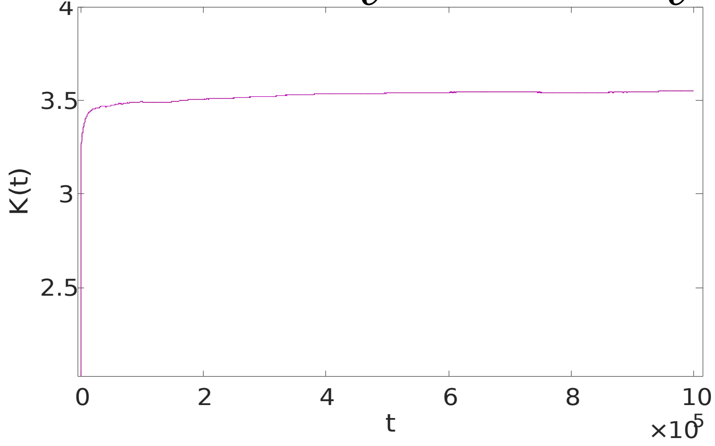
$$\mathcal{K} = \frac{\langle |x(t)|^4 \rangle}{\langle |x(t)|^2 \rangle^2} = 3.54 \pm 0.01$$

we choose GGD: $\mathcal{F}(u) \propto exp(-|u|^p)$

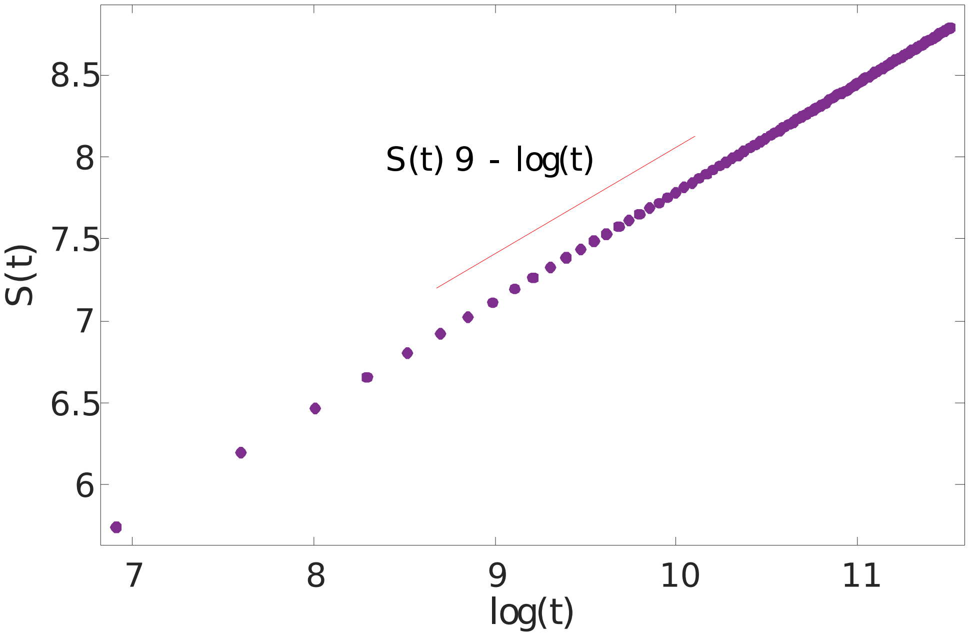
$$\mathcal{K} = \frac{\Gamma(5/p)\Gamma(1/p)}{\Gamma^2(3/p)} \Longrightarrow p = 1.607 \pm 0.006$$

$$\mathcal{P}(x,t) \sim \frac{1}{t^{\beta}} exp(-|\frac{1}{t^{\beta}}|^p)$$





DEA

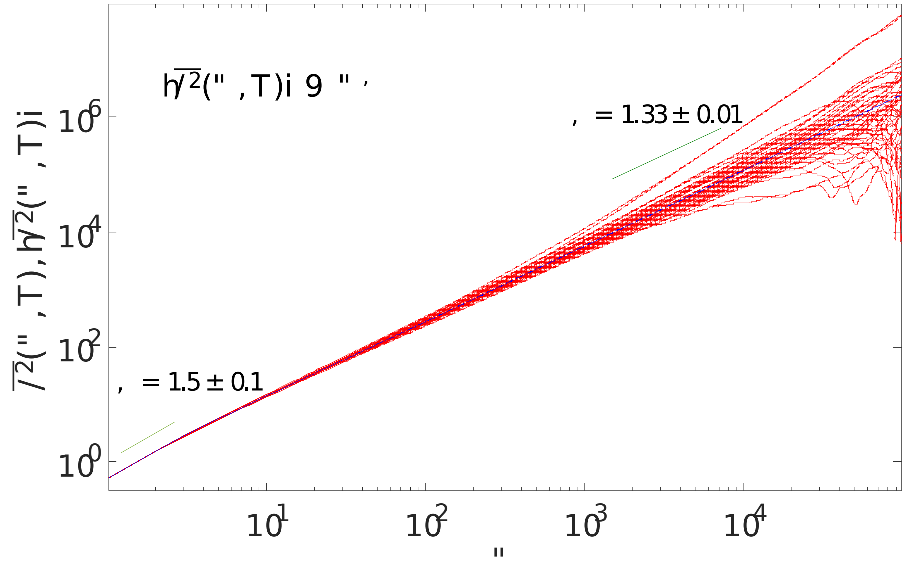


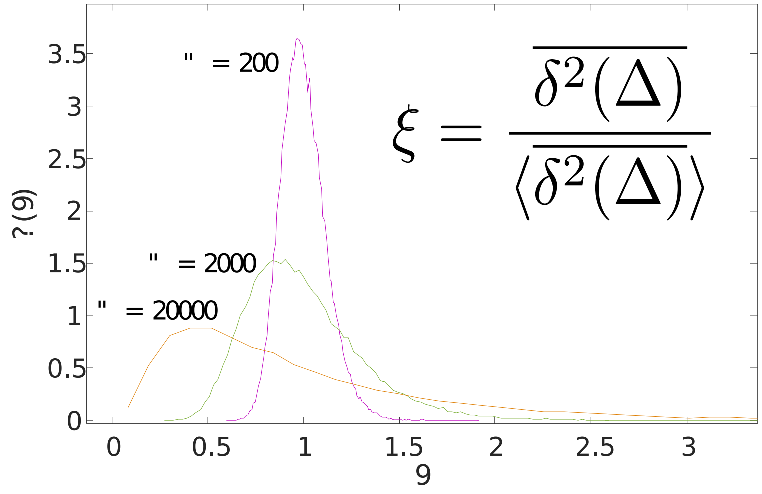
$$S(t) = -\int \mathcal{P}(\overrightarrow{r}, t) \log \mathcal{P}(\overrightarrow{r}, t) d\overrightarrow{r}$$

$$S(t) = \beta \log t - \int \mathcal{F}(u) \log \mathcal{F}(u) du = \beta \log t + cnst.$$

$$\beta = 0.6629 \pm 0.0001$$

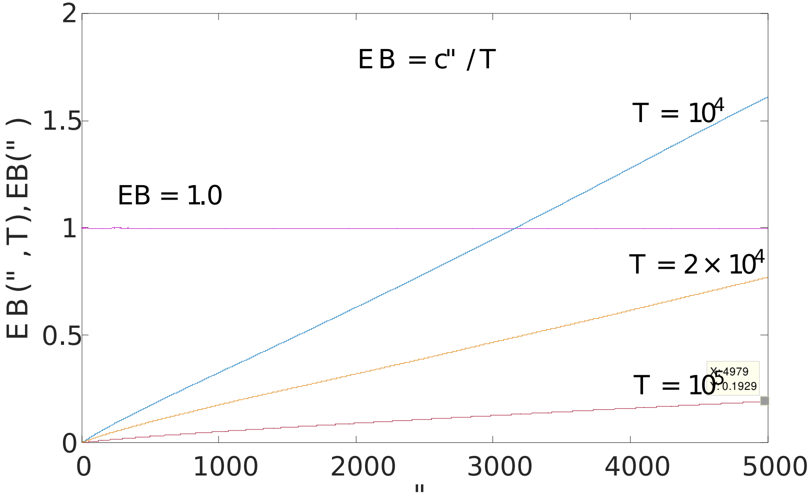
Weak Ergodicity Breaking and Aging





$$\xi = \frac{\overline{\delta^2(\Delta)}}{\langle\overline{\delta^2(\Delta)}\rangle}$$

$$\overline{\delta^2(\Delta,T)} = \frac{1}{t-\Delta}\int_0^{T-\Delta}[\overrightarrow{r}(t+\Delta)-\overrightarrow{r}(t)]^2 dt$$



$$\lim_{T\to\infty}\overline{\delta^2(\Delta,T)} = \langle\overrightarrow{r}^2(\Delta)\rangle$$

$$EB(\Delta) = \lim_{T/\Delta\to\infty}\langle\xi^2\rangle - 1$$

$$\langle\overline{\delta^2(\Delta,T)}\rangle = \frac{1}{N}\sum_{i=1}^{N}\overline{\delta^2(\Delta,T)}$$

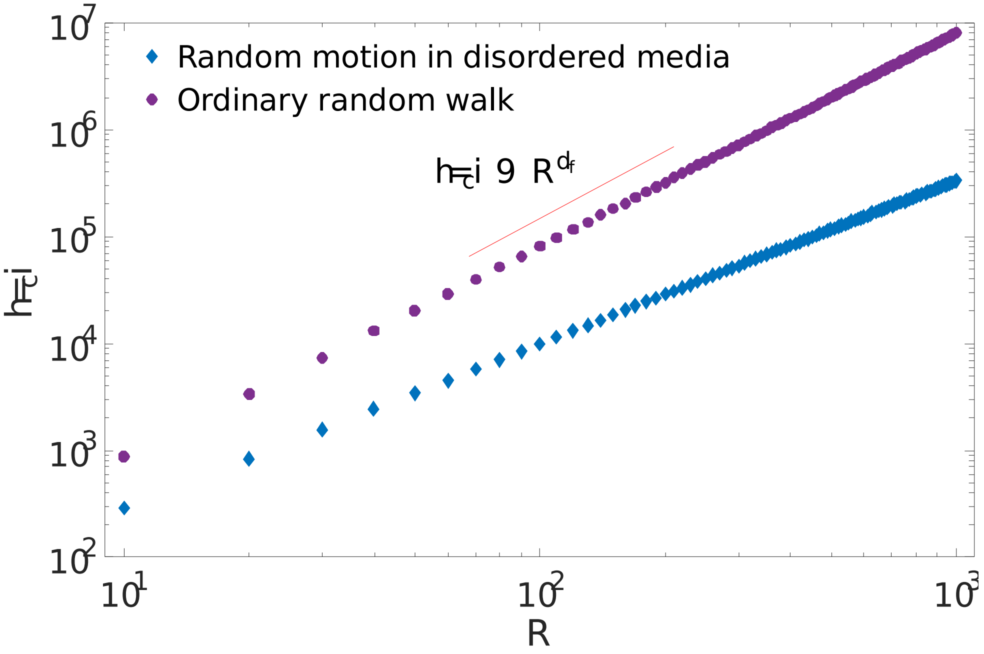
$$\mathcal{EB} = \frac{\langle\overline{\delta^2(\Delta)}\rangle}{\langle\overrightarrow{r}^2(\Delta)\rangle}$$

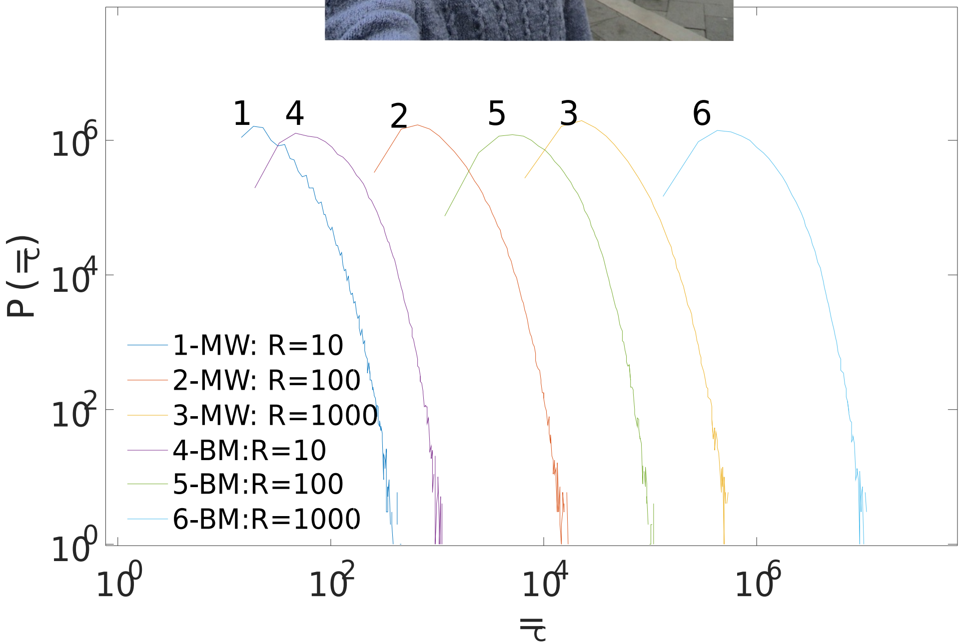
First Exit Time

$$\tau_c = inf\{t > 0 : |\overrightarrow{r}(t)| \geq R\}$$

$$\langle \tau_c \rangle \sim R^{d_f}$$

$$d_f = 1.522 \pm 0.005$$
$$d_f = 1.998 \pm 0.005$$





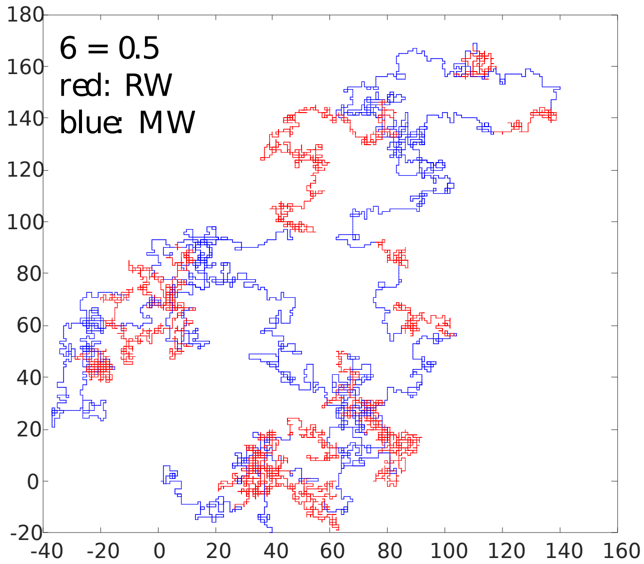
Random Dance

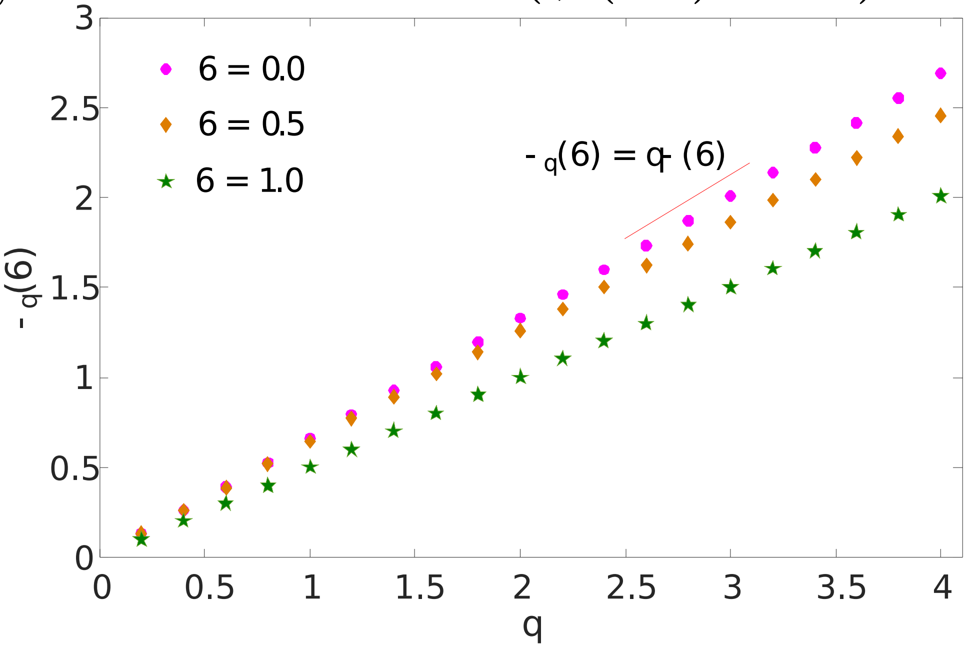
$$x(t+\Delta t) = x(t) + \Big[\phi(\tau_n)\xi(t) + \Big(1-\phi(\tau_n)\Big)\Big\{\eta(t)\mathcal{U}(y(t))\Big\}\Big]\Delta t$$

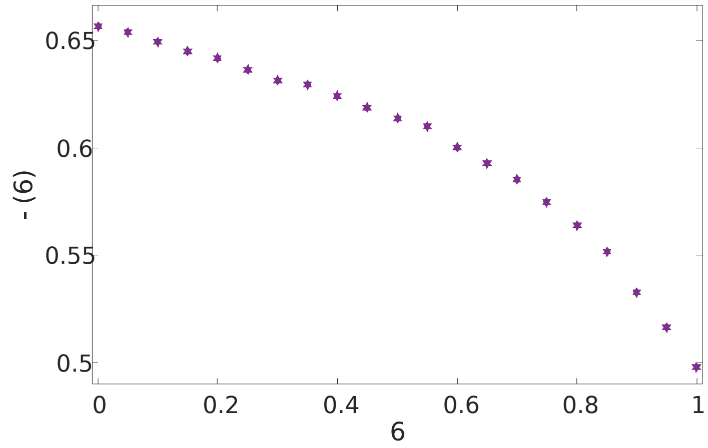
$$y(t+\Delta t) = y(t) + \Big[\phi(\tau_n)\xi(t) + \Big(1-\phi(\tau_n)\Big)\Big\{\Big(1-\eta(t)\Big)\mathcal{V}(x(t))\Big\}\Big]\Delta t$$

$$\sum_n \tau_n = T$$

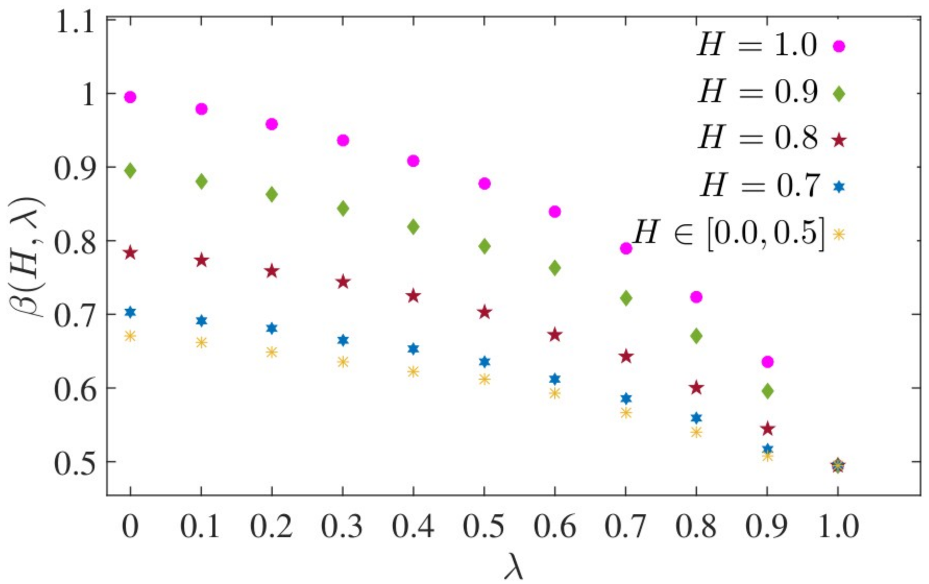
$$P(\phi(\tau_n)=1) = \lambda$$
$$P(\phi(\tau_n)=0) = 1-\lambda$$

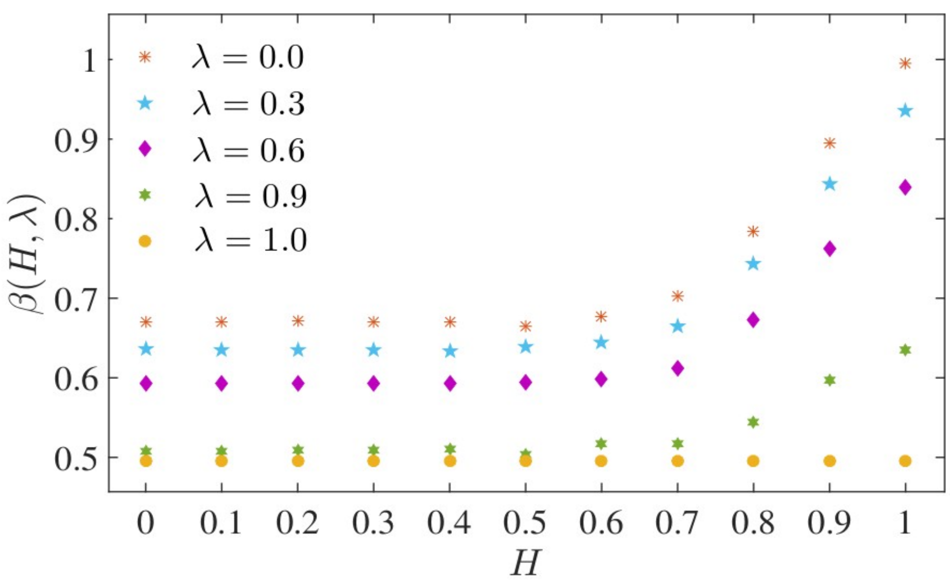






Correlation Effect



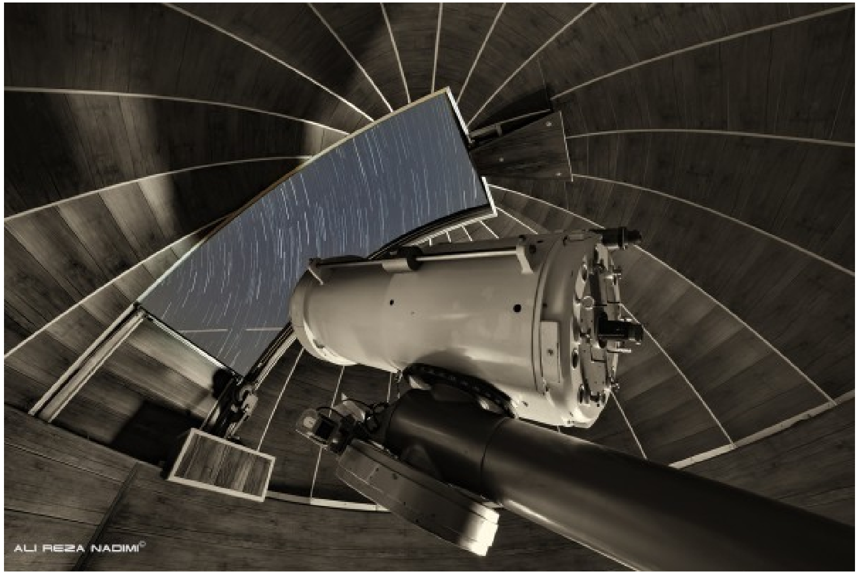


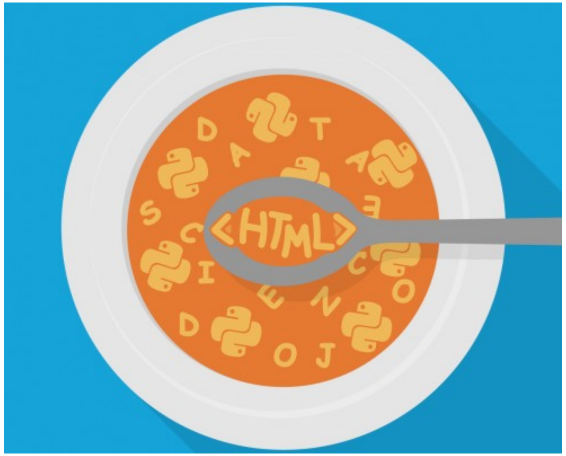
$$\langle \mathcal{U}_x[y_1]\mathcal{U}_x[y_2]\rangle = \sigma^2[H(2H-1)]|y_1 - y_2|^{2H-2}$$

$$\langle \mathcal{U}_y[x_1]\mathcal{U}_y[x_2]\rangle = \sigma^2[H(2H-1)]|x_1 - x_2|^{2H-2}$$

Previous Projects

- Synchronization
- Variable Star Photometry
- Internship as Data Scientist





Current Project